A Lloyd-Max based quantizer of L-values for AWGN and Rayleigh Fading channel

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***Abstract*—L-values have been widely adopted due to their outstanding features in many scenarios of digital communication systems, for instance, in channel decoders or in cooperative relays, where in the latter L-values could be exchanged between physically separated relays. However, an L-value is a real number, and for this reason, it must be quantized before being processed in any finite state machine. This irreversible process reduces the mutual information contained implicitly in the L-value. Therefore the quantizer must be designed carefully to jeopardize the mutual information contained in the L-value as little as possible. In literature, solutions for such a quantizer are proposed only for systems considering AWGN channels. In this paper a new method for the design of a quantizer of L-values is proposed which allows us to consider systems with both AWGN and flat Rayleigh fading channels. To this end, an optimized Lloyd-Max based quantizer of L-values is developed. It is shown that the cost function suitable for the Lloyd-Max algorithm can be designed in terms of the information loss instead of the mutual information.**

***Index Terms*—LLR quantizer, L-values, Lloyd-Max quantizer, Mutual Information**

I. Introduction

Soft information, such as Log-Likelihood Ratio (LLR) or L-values [1], has been very useful in improving the performance of channel coding in wireless communication systems. Also in the field of cooperation, e.g., cooperative receivers and cooperative relays, some schemes require the exchange of L-values between computers. Therefore, L-values must be quantized before any further processing stage in wireless communication systems. Minimizing the mutual information loss due to quantization given a certain compression rate is the key point of quantizer design for L-values.

In prior studies, a quantizer for L-value is designed by minimizing the expectation $\mathrm{E}[(L-\hat{L})^2]$, L denotes the L-value and $\hat{L}$ refers to its quantzied version, assuming the L-value as an independent variable [2], [3] only when an AWGN channel is considered. Indeed, L-values at the receiver are generated in terms of the conditional probabilities of the system. Therefore, such a quantizer optimizes the distortion of the L-values but neglects the information conveyed by the source, which is implicitly contained in it but in a non-linear fashion. Some other approaches take advantage of this non-linearity feature in designing the quantizer for the L-values [2], [4]. They make use of other non-linear functions with similar saturation behavior as the mutual information has with respect to L-values. Moreover, these quantizers are designed only for L-values at low magnitudes in the interval $[-a, a]$, where $\mid a \mid \ll \infty$ is an arbitrary number. Therefore, these quantizers do not optimally preserve the mutual information contained in the L-values and give only a local solution.

A more complex method is given in [5], where a quantizer for L-values with the constraint of maximizing the mutual information is designed. This method employs a steepest ascent technique and strictly applies only to systems with AWGN channels [5]. A new method is required for the case when a flat Rayleigh fading channel is assumed.

In this paper, a less complex scheme is proposed which is different from prior schemes. The aim is to design a Lloyd-Max based quantizer with capabilities to minimize the information loss due to the quantization process; this is equivalent to maximizing the mutual information between the source and the receiver after the quantization is accomplished. One of the advantages of the Lloyd-Max quantizer [6], [7] is its simplicity, easy of implementation. The main contribution of the proposed scheme is the design of a quantizer of L-values dealing not only with AWGN channels but also with Rayleigh fading channels. We also extend this method to the special case when the channel state information (CSI) is ignored; however, when this information is available the scheme can easily be incorporated. Analytical and experimental results show that the proposed scheme outperforms the classical method which is designed by minimizing the expectation $\mathrm{E}[(L-\hat{L})^2]$.

The paper is structured as follows. In Section II, the Lloyd-Max quantizer and some of its important features are presented. Sections III and IV give an introduction of L-values and mutual information respectively. Afterwards, Section V is dedicated to the proposed design of the quantizer. Numerical results and performance comparisons for illustration are presented in Section VI which is followed by a conclusion in Section VII.

II. Lloyd-Max Quantizer

Given a set Y, an input value $y \in Y$ and a set of discrete values $\hat{Y} = \{ \hat{y}_j \}_{j=1}^{N}$, an optimized quantizer

$$Q : \begin{cases} Y & \to \hat{Y} \\ y & \mapsto \arg\min_{\hat{y}_j} \, (y-\hat{y}_j)^2 \end{cases} \tag{1}$$

minimizes the mean-square quantization error (distortion)

$$D = \sum_{j=1}^{N} \int_{\Re_j} (y - \hat{y}_j)^2 \, p(y) \, dy \qquad (2)$$

as much as possible by optimally selecting the output levels $\{\hat{y}_j\}_{j=1}^N$ and the corresponding input ranges $\{\Re_j\}_{j=1}^N$ with $N = 2^b \ll \infty$ quantization levels for a given compression rate b in [bits/symbol], where $\Re_j = (a_{j-1}, a_j]$ for $j \in \{1, 2, ..., N\}$ with $a_0 = \inf\{Y\}$ and $a_N = \sup\{Y\}$; $p(y)$ is the probability distribution of the input y.

The optimized quantized values of a Lloyd-Max quantizer are estimated by setting $\frac{\partial D}{\partial \hat{y}} = 0$ and solving for $\hat{y}_j$; this gives

$$\hat{y}_j \quad = \quad \frac{\int_{\Re_j} y \, p(y) \, dy}{\int_{\Re_j} p(y) \, dy}. \qquad (3)$$

Similarly, the boundaries of the optimized quantization regions can also be found by setting $\frac{\partial D}{\partial a_j} = 0$ and solving for a_j. For $\hat{y}_{j+1} \neq \hat{y}_j$, this is

$$a_j \quad = \quad \frac{\hat{y}_{j+1} - \hat{y}_j}{2}. \qquad (4)$$

The quantized value in (3) is computed as the conditional mean of its region, i.e., $\hat{y}_j = \mathrm{E}[y]$ when $y \in \Re_j$. The boundaries for the quantization regions $\{a_j\}_{j=1}^{N-1}$ are computed by means of (4) to be the midpoint of the quantized values, or in other words, to be the arithmetic average of the two neighbouring quantized values.

To solve (3) and (4), a total of $2N - 1$ variables have to be found numerically, for which the Lloyd-Max algorithm is suitable. The convergence of the algorithm has been proved for a log-concave distribution $p(y)$, $\int_{\Re_j} p(y) \, dy > 0, \forall j$ and $\int_{a_0}^{a_N} p(y) \, dy = 1$. In addition, the distortion function (2) must also be smooth. Detailed proofs for convergence and for the applicability of the algorithm can be found, for example, in [8], [9] as well as in the original papers by Max [7] and Lloyd [6].

III. L-values Distribution

Given a communications system $y = ax + n$ where a is the normalized Rayleigh fading factor, $x \in \{1, -1\}$ is the transmitted symbol with probabilities $\mathrm{P}(x)$ and power $\sigma_x^2 = \frac{E_s}{T_s} = 1$, where E_s and T_s are the signal energy and sampling period respectively; $n \sim \mathcal{N}(0, \sigma_n^2)$ is the additive noise of the channel with variance $\sigma_n^2 = \frac{N_0}{2T_s}$, the complementary a-posteriori (APP) L-value is defined [10] by

$$\begin{aligned} \mathrm{L}(\hat{x} = x|y) \quad &= \quad \ln\left(\frac{\mathrm{P}(x = +1|y)}{\mathrm{P}(x = -1|y)}\right) \\ &= \quad 4a\frac{E_\mathrm{s}}{N_0} y + \mathrm{L}(x) \\ &= \quad L_\mathrm{c} y + \mathrm{L}(x), \end{aligned} \qquad (5)$$

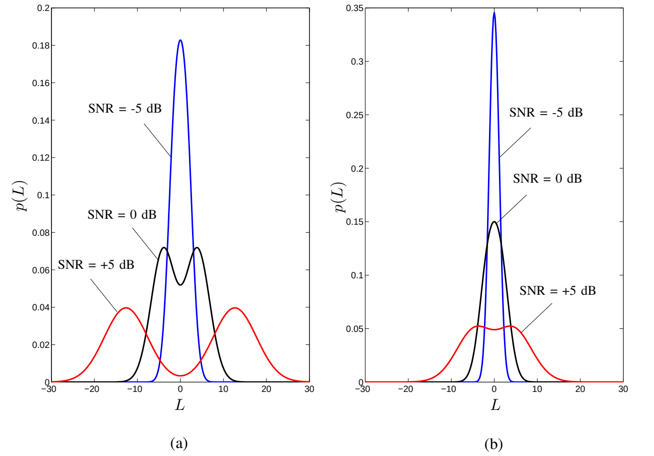


Fig. 1. Probability distribution of L-values for (a) AWGN channel given in (7), and (b) Rayleigh fading channel given in (9).

where $\hat{x}$ is the estimated symbol at the receiver, $L_\mathrm{c} y = \mathrm{L}(y|x = \pm 1)$ is the soft output of the channel and $\mathrm{L}(x)$ corresponds to the a-priori L-value at the source. Assuming equally likely symbols x, the a-priori L-value is $\mathrm{L}(x) = 0$ and therefore $\mathrm{L}(\hat{x} = x|y) = \mathrm{L}(y|x = \pm 1)$. If no misunderstanding arises, hereafter $\mathrm{L}(y|x)$ will be referred to as just "L" for simplicity of notation.

At the receiver, the sign of L corresponds to the hard decision and its magnitude to the measure of its reliability. This means that the probability of making a correct estimation can be obtained in terms of $|L|$ [1], which is

$$\mathrm{P}(\hat{x} = x|L) = \frac{e^{|L|}}{1 + e^{|L|}}. \qquad (6)$$

Finally, it can be noticed that the distribution of the L-value [10] given in (5) is a superposition of two Gaussian distributions $\mathcal{N}(\pm\frac{1}{2}\sigma_\mathrm{L}^2, \sigma_\mathrm{L}^2)$ where $\sigma_\mathrm{L}^2 = 2aL_\mathrm{c}$. Nevertheless, the distribution for AWGN channels and the distribution for Rayleigh fading channels have different behaviors. These distributions are essential for the quantizer design, and therefore they are described in the next sections.

A. L-values Distribution for AWGN Channel

Assuming $a = 1$, the mean $\mathrm{E}[\mathrm{L}(y|x)] = \mu_\mathrm{L} = 2/\sigma_n^2$ is the half of the variance $\mathrm{var}(\mathrm{L}(y|x)) = \sigma_\mathrm{L}^2 = 4/\sigma_n^2$; the distribution $p(L)$ is expressed as

$$p(L) = \frac{1}{2} \frac{1}{\sqrt{2\pi\sigma_\mathrm{L}^2}} \left(e^{-\frac{(L+\sigma_\mathrm{L}^2/2)^2}{2\sigma_\mathrm{L}^2}} + e^{-\frac{(L-\sigma_\mathrm{L}^2/2)^2}{2\sigma_\mathrm{L}^2}} \right). \qquad (7)$$

The distribution given in (7) is illustrated in Figure 1. It depends on both the signal-to-noise ratio (SNR) of the channel and the L-value. Moreover, a superposition of two Gaussian distributions and the corresponding relation between the means and the variances can be observed.

B. L-values Distribution for Flat Rayleigh Fading Channel

For the case of flat Rayleigh fading channel, the fading amplitude in (5) has a density function $p(a) = 2a\exp(-a^2)$ with $\mathrm{E}[a^2] = 1$. The distribution of the L-values with ideal channel state information (CSI) is

$$p(L) = \frac{\sigma_n}{\sqrt{8\pi}} \int_0^{\infty} \left(\mathrm{e}^{-\frac{\left(L-\xi a^2\right)^2}{4\xi a^2}} + \mathrm{e}^{-\frac{\left(L+\xi a^2\right)^2}{4\xi a^2}} \right) \mathrm{e}^{-a^2} da, \quad (8)$$

with $\xi = 2/\sigma_n^2$. This distribution is clearly dependent on a. In this case, a different quantizer/dequantizer should be designed for each possible value of a thereby occupying additionally resources of the system, e.g., memory. In addition, the CSI could also not be available on all the stages of a communication system. Therefore, when no CSI is available, the distribution of L-values must be approximated [11] by considering $\mathrm{E}[a] = \int_a a \; p(a) \; da = 0.8862$. This leads to an approximated APP L-value $\mathrm{L}_c y \approx \frac{2}{\sigma_n^2} y \mathrm{E}[a]$. With this approximation, the distribution of L-values becomes

$$\begin{aligned} p(L) =& \frac{\sigma_n \Delta^2}{4\mathrm{E}[a]} \mathrm{e}^{\left(-\frac{\Delta^2 \sigma_n^2 L^2}{4(\mathrm{E}[a])^2}\right)} \left[\sqrt{\frac{8}{\pi}} \, \mathrm{e}^{\left(-\frac{\Delta^2 L^2}{8(\mathrm{E}[a])^2}\right)} \right. \\ &\left. + \frac{\Delta L}{2\mathrm{E}[a]} \mathrm{erfc}\left(\frac{-\Delta L}{\sqrt{8}\mathrm{E}[a]}\right) - \frac{\Delta L}{2\mathrm{E}[a]} \mathrm{erfc}\left(\frac{\Delta L}{\sqrt{8}\mathrm{E}[a]}\right) \right], \end{aligned} \quad (9)$$

where $\Delta = \sqrt{\sigma_n^2/(2\sigma_n^2+1)}$. The density $p(L)$ in (9) is illustrated in Figure 1. Unlike the distribution of L-values for AWGN channels, the distribution of L-values for flat Rayleigh fading channels is denser at lower $|L|$ and is less susceptible to $|L|$.

IV. Mutual Information

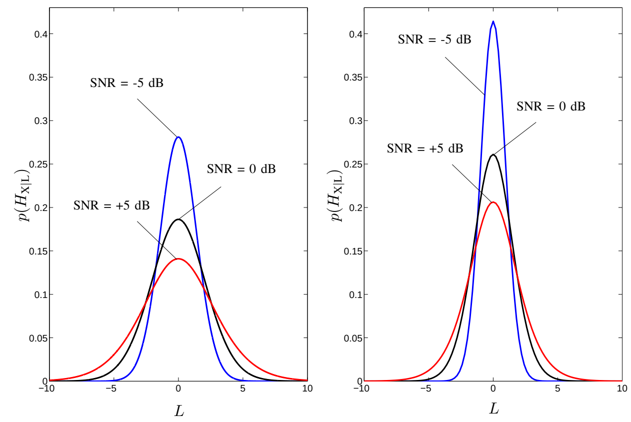
In a binary communication system, a source X with symbols $x \in \{-1, 1\}$ and probabilities $\mathrm{P}(x = \pm 1)$ conveys information to a receiver. $L \in \mathbb{R}$ is the APP L-value determined at the receiver in (5). The mutual information $\mathrm{I}(X; L)$ measures how much common information is contained in both X and L. For this system, a general expression of the mutual information [12] is

$$\mathrm{I}(X; L) = \mathrm{H}(X) - \mathrm{H}(X|L). \quad (10)$$

The first term on the right-hand side of (10) corresponds to the entropy of the source X, which for a binary source with equally distributed symbols is $\mathrm{H}(X) = 1$. The second term measures the information conveyed by the source which does not reach the receiver. Thus, the mutual information in (10) can be writen as

$$\mathrm{I}(X; L) = 1 - \mathrm{H}(X|L) \quad (11)$$

and the information loss $\mathrm{H}(X|L)$ is determined in terms of the probabilities of the system by



(a) (b)

Fig. 2. Probability density function of the information loss $p(H_{\mathrm{X|L}})$, assuming (a) AWGN channel and (b) Rayleigh fading channel without CSI.

$$\mathrm{H}(X|L) = \sum_{x \in \{+1,-1\}} \int_{-\infty}^{+\infty} p(x, L) \log_2 \frac{1}{p(x|L)} \, dL, \quad (12)$$

where $p(x, L)$ is the joint probability between x and L; $p(x|L)$ denotes the conditional probabilities of the transmission. As indicated by (11), reducing the information loss $\mathrm{H}(X|L)$ increases the mutual information $\mathrm{I}(X; L)$. Meaning thus that the quantizer can be designed either to maximize $\mathrm{I}(X; L)$ or to minimize $\mathrm{H}(X|L)$. This paper investigates the last option.

V. Quantizer Design

A. Quantizer Design for L-values

A Lloyd-Max quantizer for L-values is straightforward to design. Examples for AWGN channel can be found in [2], [3]. The quantized L-values $\{\hat{L}_j\}_{j=1}^{N}$ and the boundaries for the quantization regions $\{a_j\}_{j=1}^{N-1}$ are optimally chosen by means of (3) and (4) respectively. For a general optimized solution, the lower and upper boundaries of the quantizer input muss be set to $a_0 = -\infty$ and $a_N = +\infty$. For AWGN channels and Rayleigh fading channels, with or without CSI, the distributions to consider in (3) are given in Section III. These distributions satisfy the necessary conditions of Section II for applying Lloyd-Max algorithm. In other words, this quantizer is designed in such a way that $\mathrm{I}(L; \hat{L})$ is maximized for a given compression rate b.

B. Quantizer Design Minimizing the Information Loss

The goal of this paper, it to design a quantizer capable of reducing the mutual information loss due to the quantization noise

$$\mathrm{I_Q}(X; L) = \mathrm{I}(X; L) - \mathrm{I}(X; \hat{L}), \quad (13)$$

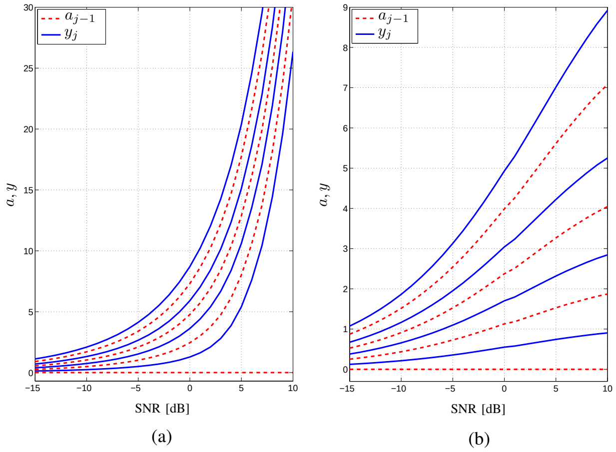


Fig. 3. Optimum quantized values $|y_j|$ and quantization boundaries $|a_{j-1}|$ [13], for $N = 8$ and AWGN channel, regarding the distribution of L-value $p(L)$ in (a) and the distribution of the information loss $p(H_{\mathrm{X|L}})$ in (b).

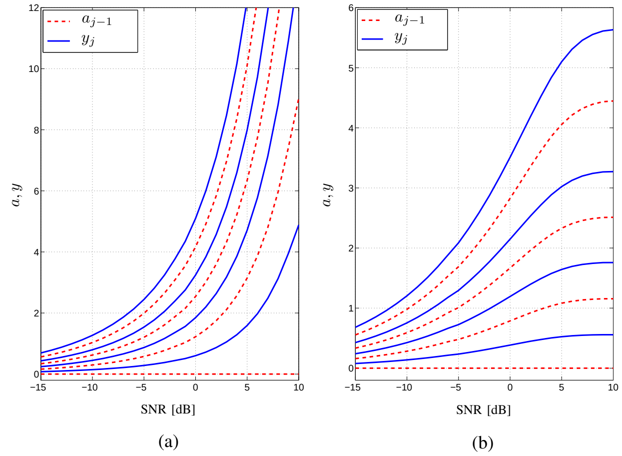


Fig. 4. Optimum quantized values $|y_j|$ and quantization boundaries $|a_{j-1}|$ [13], for $N = 8$ and Rayleigh fading channel without CSI, regarding (a) the distribution of L-value $p(L)$ and (b) the distribution of the information loss $p(H_{\mathrm{X|L}})$.

with $\hat{L} = Q(L)$ as the quantized L-value. The mutual information $\mathrm{I}(X;L)$ is defined in (10) and the mutual information after quantizing $\mathrm{I}(X;\hat{L})$ is given by

$$\mathrm{I}(X;\hat{L}) = \mathrm{H}(X) - \mathrm{H}(X|\hat{L}). \quad (14)$$

Assuming $\mathrm{H}(X) = 1$ in (14), it can be seen that minimizing $\mathrm{I_Q}(X;L)$ in (13) is equivalent to minimizing $\mathrm{H}(X|\hat{L})$ in (14). Hence, to minimize the distortion of the mutual information is equivalent to minimize the distortion

$$D = \min_{\{\hat{L}_j\}_{j=1}^{N}, \{a_j\}_{j=1}^{N-1}} \mathrm{E}[(\mathrm{H}(X|L) - \mathrm{H}(X|\hat{L}))^2], \quad (15)$$

with respect to $\hat{L}$ by means of (2). Then, in order to implement the Lloyd-Max quantizer introduced in Section II, we derive the distribution of $\mathrm{H}(X|L)$ as follows.

The source X transmits the symbol $x \in \{\mathrm{X}_i\}_{i=0}^{1}$ to the receiver with a-priori probabilities $\mathrm{P}(\mathrm{X}_i)$. The distribution $p(L)$ of an L-value at the receiver is given in Section III for AWGN and Rayleigh fading channel with and without CSI. Without any loss of generality, X_0 is assumed to be transmitted. The conditional probability of a transmission error is $\mathrm{P}(\mathrm{X}_1|L) = P_{\mathrm{e}}$ and the conditional probability of an error-free transmission is $\mathrm{P}(\mathrm{X}_0|L) = 1 - P_{\mathrm{e}}$, where P_{e} is determined via (6), i.e., $P_{\mathrm{e}} = 1 - \mathrm{P}(\hat{x} = x|L)$. Further, the probabilities of an erroneous reception and an error-free reception are $\mathrm{P}(\mathrm{X}_0, L) = (1 - P_{\mathrm{e}})p(L)$ and $\mathrm{P}(\mathrm{X}_1, L) = (P_{\mathrm{e}})p(L)$ respectively. Substituting these probabilities in (12) gives

$$\mathrm{H}(\mathrm{X}|L) = \int_{-\infty}^{+\infty} (1 - P_{\mathrm{e}})p(L)\log_2 \frac{1}{1 - P_{\mathrm{e}}}\, dL + \int_{-\infty}^{+\infty} P_{\mathrm{e}}p(L)\log_2 \frac{1}{P_{\mathrm{e}}}\, dL, \quad (16)$$

and after factorizing $p(L)$ and normalizing the average information loss, the distribution of $\mathrm{H}(X|L)$ becomes

$$p(H_{\mathrm{X|L}}) = \frac{p(L)H_{\mathrm{L}}}{\int_{-\infty}^{+\infty} p(L)H_{\mathrm{L}}\, dL}, \quad (17)$$

where $H_{\mathrm{X|L}} := \mathrm{H}(X|L)$, just for simplicity of notation. $H_{\mathrm{L}} = \mathrm{H}(P_{\mathrm{e}})$ denotes the entropy of the conditional probability of an error transmission [10]. The distribution given in (17) is depicted in Figure 2 for AWGN and Rayleigh fading channel without CSI. It can be noted that in contrast to $p(L)$ depicted in Figure 1, $p(H_{\mathrm{X|L}})$ is barely dependent on the quality of the channel. Additionally, it is denser around the unreliable L-values, i.e., at the region where the mutual information is lower (high entropy). Lastly, it is also noteworthy that the

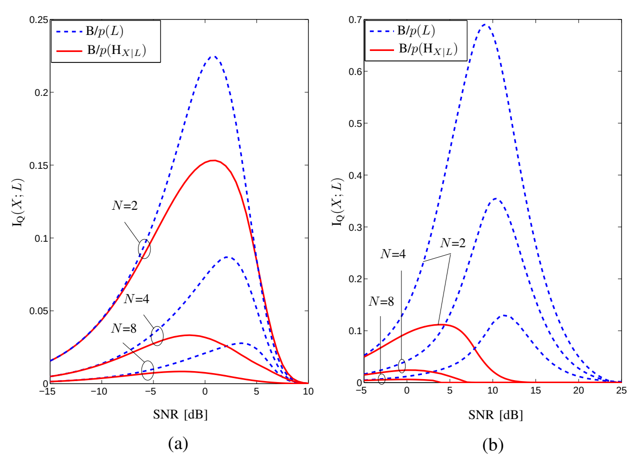


Fig. 5. Performance comparison of quantizers based on $p(L)$ and $p(H_{\mathrm{X|L}})$ for $N \in \{2, 4, 8\}$ [13]. The plots correspond to the mutual information loss $\mathrm{I_Q}(X;L)$ given in (13) considering (a) AWGN channel and (b) Rayleigh fading channel.

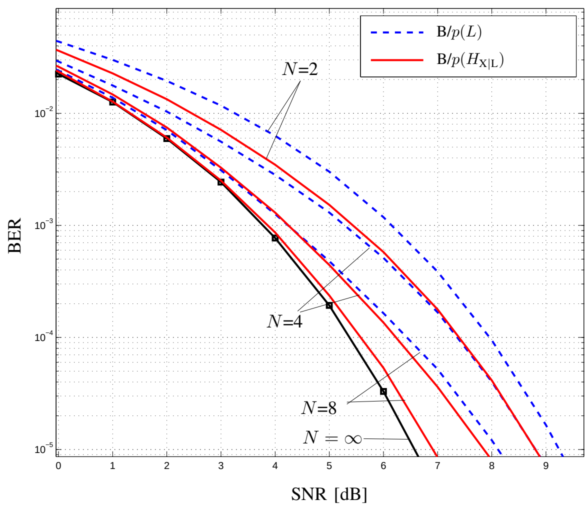


Fig. 6. The resulting BER comparison after combining L-values between two cooperative receivers assuming AWGN channel [13], which is performed by adding the non-quantized and quantized L-values using $N = \{2, 4, 8\}$ level quantizer based on $p(L)$ and $p(H_{X|L})$.

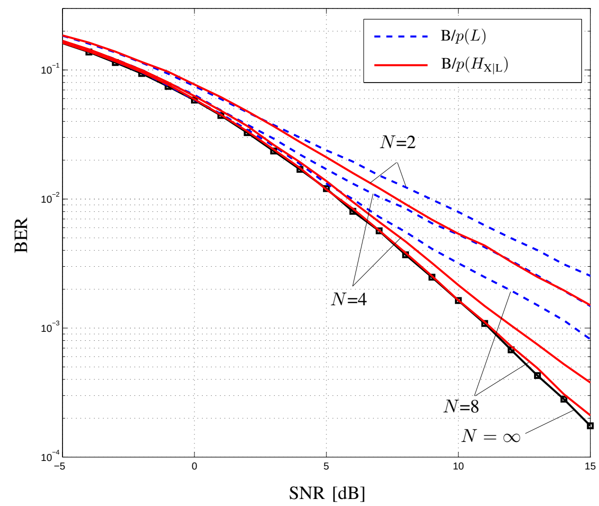


Fig. 7. The resulting BER comparison after combining L-values between two cooperative receivers assuming Rayleigh fading channel [13], which is performed by adding the non-quantized and quantized L-values using $N = \{2, 4, 8\}$ level quantizer based on $p(L)$ and $p(H_{X|L})$.

information loss is inversely proportional to the SNR and to $|L|$.

Finally, the Lloyd-Max quantizer for L-values can be designed by minimizing the mutual information distortion given a compression rate b. With (3) and (4), $\{\hat{L}_j\}_{j=1}^{N}$ and $\{a_j\}_{j=1}^{N-1}$ are optimally chosen. Due to $L \in \mathbb{R}$, the boundaries $a_0 = -\infty$ and $a_N = +\infty$ are deduced. The distribution in (17) fulfills the conditions in Section II required for the convergence of the Lloyd-Max algorithm.

VI. Results

In this section, we apply the proposed scheme and present the results of the design of two quantizers for L-values with their corresponding performances, in terms of mutual information loss and bit error rate (BER).

A. Quantizers Design

In Section V the designs of two quantizers were detailed. Both are based on the Lloyd-Max quantizer described in Section II. The difference between them is the optimization criteria, i.e., the distributions used in (2). For the quantizer that optimizes L-values (Section V-A), the distribution implemented is $p(L)$. In the same manner, the distribution $p(H_{X|L})$ is considered for the quantizer that minimizes the information loss (Section V-B). Both quantizers are compared to measure the performance of the proposed strategy. As an example, the results of these quantizers with $N = 8$ are illustrated in Figures 3 and 4 for AWGN and Rayleigh fading channel without CSI respectively. Only the positive quantized values and quantization boundaries for any SNR in [-15,10] are presented; the negative values can be obtained by symmetry. The different limits of the vertical axes between (a) and (b) must be noticed in each figure. In (a), the quantizer minimizes the expectation $\mathrm{E}[(L-\hat{L})^2]$. Conversely, in (b) the expectation $\mathrm{E}[(\,\mathrm{H}(X|L)-\mathrm{H}(X|\hat{L})\,)^2]$ is minimized, therefore, the quantized values remain near $L = 0$, where (17) is mostly concentrated. The quantizer for Rayleigh fading channel with CSI is straightforward to design by means of computing $p(L)$ in (8) for a given "a" before proceeding with the Lloyd-Max algorithm.

B. Comparison of the Mutual Information Loss

The mutual information loss for each quantizer was computed analytically. Figure 5 shows the performance of the quantizers. The mutual information loss due to the quantization noise, given in (13), is depicted in (a) for AWGN channels and in (b) for Rayleigh fading channels. Two strategies are compared. The dashed plots show the mutual information loss due to the quantizer based on $p(L)$, and the continuous plots show the mutual information loss due to the quantizer based on $p(H_{X|L})$. It is clear, that the quantizer based on $p(H_{X|L})$ performs better than the quantizer based on $p(L)$, independent of the channel considered.

C. Comparison of the Bit Error Rate

An alternative to measuring the performance of each quantizer in terms of BER is to simulate a wireless communications system with one source and two cooperative receivers. For instance, a source transmits a symbol to the receivers, and afterwards, one receiver shares its L-values with the other. In order to avoid extra cooperation time, the receiver does not share its CSI. A BPSK modulation is assumed and the channels from the source to each of the receivers are independent but with same statistical characteristics. Y_1 and Y_2 are the receivers and L_1 and L_2 their L-values respectively. The receiver Y_2 quantizes its received L-value, i.e., $\hat{L}_2 = Q(L_2)$, and cooperates with Y_1 by means of sharing its quantized L-values. In Y_1 the combination $L_q = \hat{L}_2 + L_1$ is executed and afterwards the hard decision is performed. Finally $| L_q |$ is compared with the symbol transmitted by the source in order to compute the BER of Y_1 after cooperation.

Several plots of the BER are depicted in Figures 6 and 7 for AWGN and Rayleigh fading channels respectively. The performance of the quantizers with $N = \{2, 4, 8\}$ levels is evaluated. The $N = \infty$ plot is a benchmark representing the ideal case of adding the unquantized L-values of both receivers (or quantizing with an $N = \infty$ level quantizer).

As Figure 5, Figures 6 and 7 show that the quantizers based on $p(H_{X|L})$ distribution outperform the quantizers based on $p(L)$ distribution. The (N-1)-level quantizer based on $p(H_{X|L})$ performs close to or even better than the N-level quantizer based on $p(L)$; this allows us to save at least one bit on the quantizer resolution. Furthermore, the 8-level quantizer based on $p(H_{X|L})$ performs close to the ideal $N = \infty$ case.

VII. Conclusion

In this paper, we present a scheme to design an optimum Lloyd-Max based quantizer for L-values. The quantizer minimizes the mutual information distortion due to the quantization process while the L-value is being quantized. It is shown that reducing the distortion based on the information loss is equivalent to maximizing the mutual information between the source and the receiver after the quantization process. Therefore, the cost function for the Lloyd-Max algorithm is developed in terms of minimizing the information loss. With this scheme, the quantizers are straightforwardly designed for AWGN channels as for flat Rayleigh fading channels. We have compared the performance of these two types of quantizers, which was done in terms of both mutual information loss and BER. The special case without CSI was considered, however, the case with available CSI is also straightforward to implement. The quantizer based on $p(H_{X|L})$ outperforms the quantizer based on $p(L)$ for both AWGN channels and for Rayleigh fading channels. Also worth mentioning is the fact that, for a given distortion in the sense of the rate-distortion theory the compression rate b can be further minimized by applying a source-coding algorithm to the output of the quantizer.

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